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STRUCTURE OF THE POTENTIAL BARRIER AT A METAL BOUNDARY

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In [1] the peculiarities of autoelectronic emission in an arc discharge plasma (F−P emission) connected with nonlinearity in the potential change in the precathode region were considered; it was shown that in the case where the negative electron charge density in the precathode layer can be neglected, the current density of F−P emission can differ by more than an order of magnitude from autoelectronic emission current density in a vacuum, as obtained with the Nordheim−Fowler expression. For the case where the precathode potential drop V_c is equal to the cathode work function φ, the modulus of the logarithm of the potential barrier transparency increases by 20% and the emission current density can decrease by a significant amount. An analogous change in current density can occur with other ratios between the quantities V_c and φ. In the present study we will consider these questions in greater detail for cases in which the negative space charge density cannot be neglected.

The potential distribution in the precathode region of an arc discharge can be obtained from the solution of the Poisson equation with the Langmuir−Macowan assumptions

$$\frac{dV}{dx}=\sqrt{16\pi j_i\left(\frac{MV_c^{1/2}}{2e}\right)\left\{\left(1-\frac{V}{V_c}\right)^{1/2}+q\left[\left(\frac{V}{V_c}\right)^{1/2}-1\right]\right\}}, \tag{1}$$

where M is the atomic weight of the ion, e is the charge of the electron, j_i is the ion current density at the cathode, j_e is the electron current density at the cathode, $q=(j_e/j_i)\sqrt{m/M}$.

Following [1] we will consider two cases.

1. For $V_c \geq \varphi$ the expression for potential barrier transparency Q is written in the form

$$Q_{F-P}=\int_{x_1}^{x_2}\sqrt{\varphi-e/4x-V(x)}\,dx, \tag{2}$$

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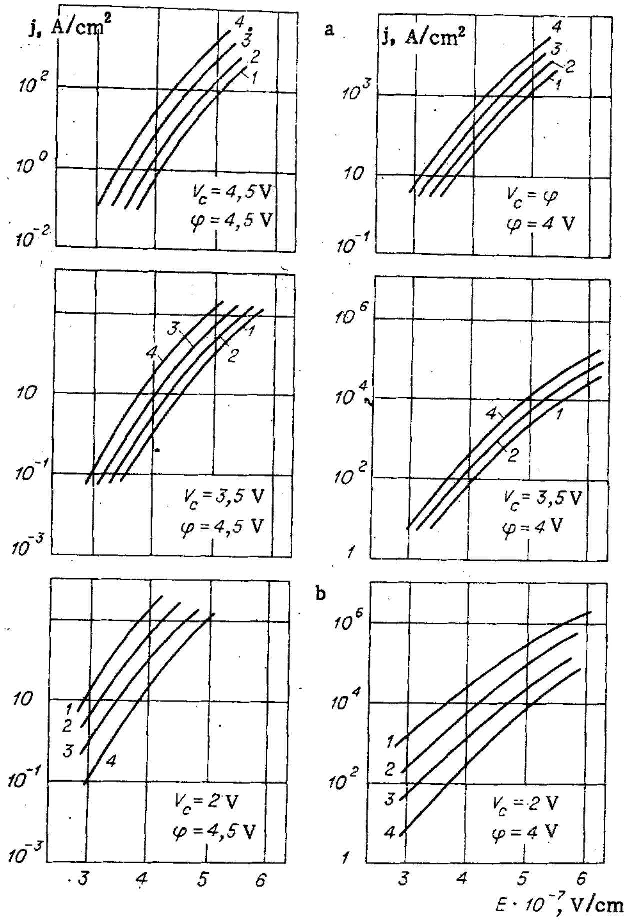

Fig. 1

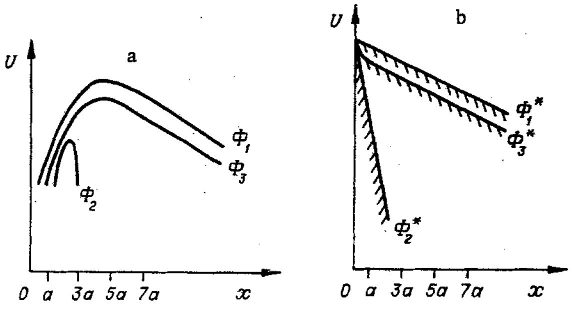

Fig. 2

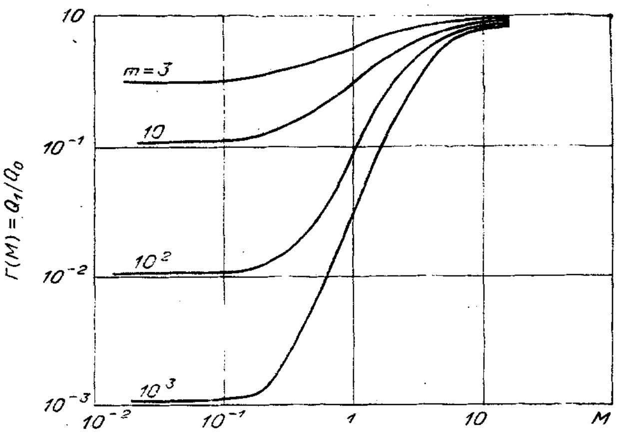

Fig. 3

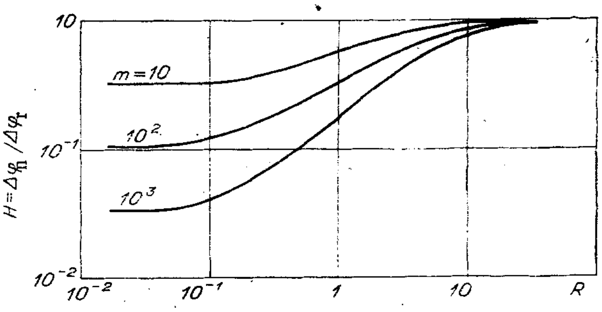

Fig. 4

where x_1 and x_2 are roots of the equation

$$\varphi - e/4x - V(x) = 0; \tag{3}$$

and V(x) is the solution of Eq. (1).

The electron emission current density passing through such a barrier can be written in the form

$$j_{F-P} = j_F \exp\,[Q_F - Q_{F-P}],$$

while for the F-emission current density at the same value of electric field intensity F_0

$$F_0 = \sqrt{16\pi j_i \left(\frac{MV_c^{1/2}}{2e}\right)[1-q]} \tag{4}$$

we calculated from Eq. (2)

$$j_F = \frac{1.55 \cdot 10^{-6} F^2}{\varphi} \exp\left[-\frac{6.85 \cdot 10^7 \varphi^{3/2}}{F}\,\theta(F)\right],$$

where θ(F) is the Nordheim function.

The dependence of j_{F-P} on F for various values of q is shown in Fig. 1a (1) q = 0, 2) q = 0.2, 3) q = 0.4, 4) F emission).

2. For $V_c < \varphi$ the expression for potential barrier transparency has the form of Eq. (2), where x_1 is the first positive (beginning with zero) root of Eq. (3), and x_2 is the value of x at $V = V_c$.

Curves of j_{F-P} vs F for various q values are shown in Fig. 1b.

We note that for $V_c \geq \varphi$ the F-P-emission current density is less than the F-emission current density and that the F-P-emission current density tends to the F-emission current density as an upper limit with increase in q. For $V_c < \varphi$ the F-P emission can be either greater or less than the F-emission current density.

Nonlinearity in the potential change in the precathode zone of an arc discharge is important for proper consideration of the so-called "cathode surface roughness (inhomogeneity) effect." The essence of this effect is that the electric field intensity at the cathode surface obtained from the Langmuir–Macowan equation (4) and substituted in the expressions for emission current density. However, in deriving Eq. (4) it was assumed that the cathode surface was planar, without roughness. In fact, as the experiments of [3] show, microinhomogeneities exist on the cathode surface, on which the electric field increases by a factor of several times. The field intensification coefficient m, presented in [3, 4], has the form

$$m = \frac{2n^3}{(1-n^2)\left(\ln\frac{1+n}{1-n} - 2n\right)}, \tag{5}$$

where n is the eccentricity of the ellipsoid that characterizes the microinhomogeneity (roughness),

$$n = \sqrt{1-(b/a)^2},$$

where *a* is the major axis and b the minor axis of the ellipsoid.

Analysis of Eq. (5) shows that m can reach a value of $\sim(10^2-10^3)$, i.e., the electric field at the cathode $F_c = (10^2-10^3)F_0$. This value of F_c is substituted in the Richardson–Schottky or Nordheim–Fowler equations to determine the density of electron emission current from the cathode.

However, the potential distribution near the microinhomogeneity in this case is severly nonlinear [4]

$$U(x) = F_0 a(1+x)\left[1 - \frac{\ln\frac{1+n+x}{1-n+x} - \frac{2n}{1+x}}{\ln\frac{1+n}{1-n} - 2n}\right]. \tag{6}$$

The structure of the potential barrier now changes sharply, and to calculate the electron emission current density we must substitute in the expression for potential barrier transparency the quantity U(x), and not $F_c = mF_0$.

We will consider the consequences to which such a change in barrier structure may lead.

According to [2], the expression for current density of autoelectronic emission has the form

$$j_F = AF^2 \exp\,[-2Q_0],$$

where

$$Q_0 = \int_{x_1}^{x_2} \sqrt{\varphi - e/4x - F_0 x}\, dx.$$

To determine the current density from a "rough" cathode, the value of Q must be calculated with the formula

$$Q_1 = \int_{x_1}^{x_2} \sqrt{\varphi - e/4x - U(x)}\, dx, \qquad (7)$$

where U(x) is taken in the form of Eq. (6) and not from the formula

$$Q_2 = \int_{x_1}^{x_2} \sqrt{\varphi - e/4x - mF_0 x}\, dx.$$

The potential barrier structure is shown in Fig. 2a, where

$$\begin{cases} \Phi_1 = \varphi - e/4x - F_0 x, \\ \Phi_2 = \varphi - e/4x - mF_0 x, \\ \Phi_3 = \varphi - e/4x - U(x). \end{cases} \qquad (8)$$

Figure 2b shows the potential barriers Φ_1^*, Φ_2^* and Φ_3^* with the term e/4x omitted, in order to show clearly the effect of nonlinearity in the potential change in the precathode layer

$$\begin{cases} \Phi_1^* = \varphi - F_0 x, \\ \Phi_2^* = \varphi - mF_0 x, \\ \Phi_3^* = \varphi - U(x). \end{cases} \qquad (9)$$

The transparency of barrier Φ_3^* depends on the dimensionless parameter M

$$M = \varphi / F_0 a.$$

(Below, in order to more clearly stress the difference between barriers with linear and nonlinear potential changes, we will consider the barriers of Eq. (9), not those of Eq. (8). All conclusions concerning the relationships between transparencies of these barriers will be valid to a high degree of accuracy for the barriers Φ_1, Φ_2, and Φ_3).

Depending on the value of M, the quantity Q_1 may be close to Q_0 or Q_2. Curves of the function $\Gamma(M) = Q_1/Q_0$ for various values of field intensification coefficient m are presented in Fig. 3. It is evident that if $M > 5$, then the value of Q_1 is close to Q_0, despite the fact that the electric field intensity on the cathode surface $F_C = mF_0$. For $M < 0.2$ the value of Q_1 is close to Q_2.

For $M = 1$ (potential change at a distance equal to the characteristic dimension of the inhomogeneity equal to the cathode work function) Q_1 corresponds approximately to the potential barrier transparency at an effective field intensity value at the cathode of $F_{eff} = \sqrt{m}F_0$.

For $n = 0$ (spherical inhomogeneity on cathode surface) the field intensification coefficient $m = 3$; the expression for potential change takes on the form

$$U(x) = F_0 a (1 + x) \left[1 - \frac{1}{(1+x)^3}\right],$$

and the expression for barrier transparency Q_1 can be written as

$$Q_1 = \int_{x_1}^{x_2} \sqrt{\varphi - e/4ax - F_0 a (1 + x) \left[1 - \frac{1}{(1+x)^3}\right]}\, dx.$$

In the cases of practical interest, the following situations can be realized:

1. Low Voltage Arc. In this case the precathode voltage drop $V_C \sim \varphi$, the precathode voltage drop layer thickness $d \sim 4V_C/3F_0$, and since the characteristic dimension of roughness $a \ll d$, then $M = \varphi/F_0 a \gg 1$. Despite the fact that the electric field intensity at the cathode surface $F_C = (10^2 - 10^3)\ F_0$, the autoelectronic current density will correspond to the barrier transparency at $F_C = F_0$ and the "roughness" effect will lead to no noticeable increase in emission current density.

2. Autoemission in a Vacuum. In this case the characteristic dimension of an inhomogeneity $a \sim 5 \cdot 10^{-4}$ cm, the field intensity at the emitter surface $F_0 \sim 2 \cdot 10^4$ V/cm, $M < 0.5$, and the Q_1 barrier transparency is close to Q_2.

In intermediate cases the transparency value should be calculated from Eq. (7), while $F_c = mF_0$ should not be substituted in the formula for calculation of autoelectronic emission current density.

We will now determine how the electron emission current density varies after passage over the "nonlinear" potential barrier (thermoelectronic emission).

The equation for the barrier may be written in the form

$$L(x) = \varphi - e/4ax - F_0 a(1+x)\left[1 - \frac{\ln\frac{1+n+x}{1-n+x} - \frac{2n}{1+x}}{\ln\frac{1+n}{1-n} - 2n}\right]. \tag{10}$$

The point of the barrier maximum is determined from the equation

$$e/4ax^2 - F_0 a[1-\beta(x)] + F_0 a(1+x)\beta'(x) = 0, \tag{11}$$

where

$$\beta(x) = \frac{\ln\frac{1+n+x}{1-n+x} - \frac{2n}{1+x}}{\ln\frac{1+n}{1-n} - 2n},$$

and the maximum height of the "nonlinear" potential barrier is equal to $\varphi_n = \varphi_0 - \Delta\varphi_n$, while for linear potential change the barrier height equals $\varphi_{eff} = \varphi_0 - \sqrt{meF_0}$. The effective potential barrier height φ_n depends on the dimensionless parameter

$$R = \frac{r_s}{a} = \frac{\frac{1}{2}\sqrt{\frac{e}{F_0}}}{a}.$$

For $R \ll 1$ the ratio $\Delta\varphi_n/\Delta\varphi_s = \sqrt{m}$, while for $R \gg 1$ the ratio $\Delta\varphi/\Delta\varphi_s = 1$, despite the fact that the electric field intensity at the cathode surface $F_{eff} = mF_0$ (we assume that $\Delta\varphi_s = \sqrt{eF_0}$). For $R = 1$, $\Delta\varphi_n/\Delta\varphi_s \approx m^{1/4}$.

Curves of the function $H(R) = \Delta\varphi_n/\Delta\varphi_s$ for various m values are presented in Fig. 4.

In conclusion, we note that at the present time the value of a is controlled down to 10^{-6} cm. Then for fields $F_0 < 10^4$ V/cm no local increase in field intensity on the cathode surface can lead to an increase in the emission current density of electrons which have passed above the potential barrier.

For fields $F_0 > 10^4$ V/cm the barrier height should be determined from Eqs. (10), (11) and one should not set $\Delta\varphi_{eff} = \sqrt{meF_0}$.

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